MENSURATION

BASIC CONCEPTS:

1.1 PERIMETERS AND AREAS OF PLANE FIGURES:

PERIMETER AND AREA	The perimeter of a plane figure is the total length of its boundary. The area of a plane figure is the amount of surface enclosed by its sides. (boundary) .

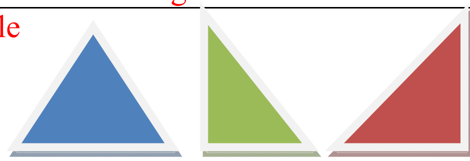
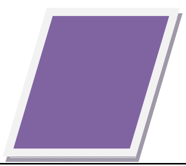
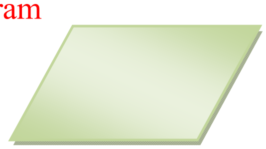
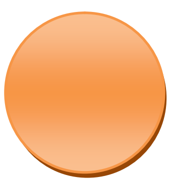
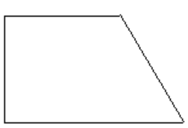
COMMON UNITS OF PERIMITER ARE METRE (m), CENTIMETRE (cm), DECIMETRE (dm) etc.

- 1 m = 100 cm and 1 cm = $\frac{1}{100}$ m = 0.01 m
- 1 m = 10 dm and 1 dm = $\frac{1}{10}$ m = 0.1 m
- 1 dm = 10 cm and 1 cm = $\frac{1}{10}$ dm = 0.1 m
- 1 m = 1000 cm and 1 mm = $\frac{1}{1000}$ m = 0.001 m and so on.

COMMON UNITS OF AREA ARE SQUARE METRE (Sq.m OR m^2), SQUARE CENTIMETRE (Sq.cm OR cm^2) , SQUARE MILLMETRE (Sq.mm OR mm^2), etc .

- 1 m^2 = 100 X 100 cm^2 = 10,000 cm^2 and 1 cm^2 = $\frac{1}{100 X 100}$ m^2 = 0.0001 m^2
- 1 m^2 = 10 X 10 dm^2 = 100 dm^2 and 1 dm^2 = $\frac{1}{10 X 10}$ m^2 = 0.01 m^2
- 1 dm^2 = 10 X 10 cm^2 = 100 cm^2 and 1 cm^2 = $\frac{1}{100}$ dm^2 = 0.01 dm^2
- 1 cm^2 = 10 X 10 mm^2 = 100 mm^2 and 1 mm^2 = $\frac{1}{100}$ cm^2 = 0.01 cm^2

The Table below gives the formula to find the perimeter/area of some figures.

Figure	Formula
Triangle 	Area = $\frac{1}{2} \times$ Base $\times$ Height $= \frac{1}{2} \times b \times h$
Square 	Perimeter = 4 $\times$ Side = 4S Area = Side $\times$ Side = S^2
Rectangle 	Perimeter = $2(l + b)$ Area = $l \times b$ l = length; b = breadth
Rhombus 	Area = Base $\times$ Height $= b \times h$
Parallelogram 	Area = Base $\times$ Height $= b \times h$
Circle 	Circumference of circle $= 2\pi r$ or πd Area of circle $= \pi r^2$ r = radius; d = diameter
Trapezium 	Area = $\frac{1}{2} \times$ Sum of parallel sides $\times$ Height $= \frac{1}{2} \times (a + b) \times h$

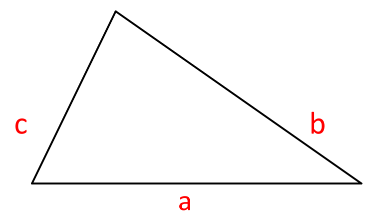
1.2 PEREMETER AND AREA OF TRIANGLES:

1. If a, b and c are the three sides of a triangle; then its

a) Perimeter = a + b + c

b) Area = $\sqrt{s\,(s-a)(s-b)(s-c)}$

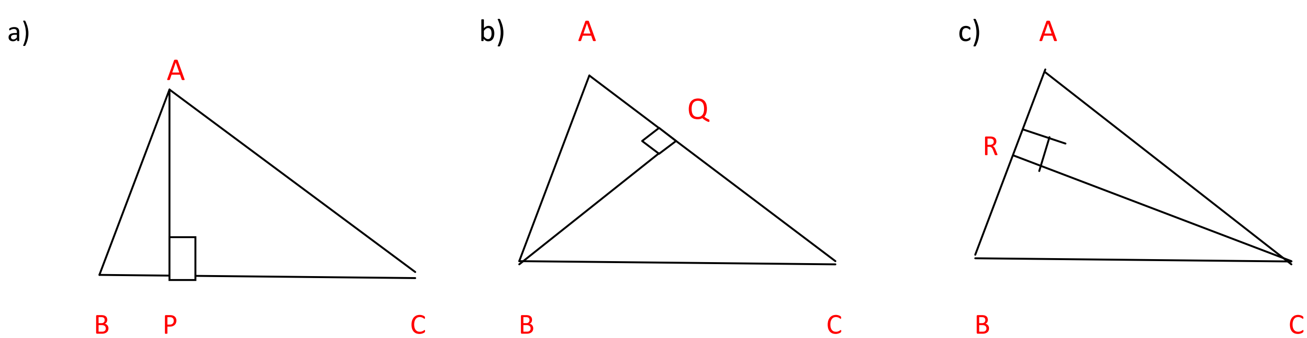
where, s = semi – perimeter of the triangle = $\frac{a+b+c}{2}$



2. If one side (base) and the corresponding height (altitude) of the triangle are known , its

Area = $\frac{1}{2}$ base X height

Any sides of the triangle can be taken as its base and the corresponding height means the length of perpendicular to this side from the opposite vertex.



If BC is taken as base area = $\frac{1}{2}$X BC X AP	If AC is taken as base area = $\frac{1}{2}$X AC X BQ	If AB is taken as base area = $\frac{1}{2}$X AB X CR

Also, Area = $\frac{1}{2}$ base X height $\Rightarrow$ a) Base = $\frac{2\ X\ Area}{Height}$

b) Height = $\frac{2\ X\ Area}{base}$

EXAMPLE1:

Find the area of a triangle whose sides are 9 cm, 12 cm and 15 cm. Also, find the length of altitude corresponding to the largest side of the triangle.

SOLUTION:

Let a = 9 cm, b = 12 cm and c = 15 cm

$\therefore$ s = $\frac{a+b+c}{2}$ = $\frac{9+12+15}{2}$ = $\frac{36}{2}$ = 18 cm.

$\therefore$ Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{18(18-9)(18-12)(18-15)}$

= $\sqrt{18 \times 9 \times 6 \times 3}$

= $\sqrt{2916}$ = 54 cm^2

Also, area of triangle = $\frac{1}{2}$ base X corresponding altitude

$\therefore$ 54 = $\frac{1}{2}$ X 15 X h [Taking largest side as the base]

$\Rightarrow$ h = $\frac{54 X 2}{215}$ cm = 7.2 cm.

EXAMPLE2:

Find the area of an equilateral triangle, whose one side is ‘a’ cm.

SOLUTION:

s = $\frac{a+b+c}{2}$ = $\frac{a+a+a}{2}$ = $\frac{3a}{2}$ [Sides of an equilateral triangle are equal]

$\therefore$ Area = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$$

$$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$$

$$= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}} = \frac{a \times a}{2 \times 2}\sqrt{3} = \frac{\sqrt{3}}{4} a^2 \, cm^2$$

Tips for Students:

The area of an equilateral triangle of side 'a unit' = $\frac{\sqrt{3}}{4}\, a^2 = \frac{\sqrt{3}}{4}(\text{side})^2$

e.g.- If the side of an equilateral triangle is 6 cm ; its area $= \frac{\sqrt{3}}{4} \text{X} (6)^2 \, cm^2 = 9\sqrt{3} \, cm^2$.

$= 9 \text{ X } 1.732 \, cm^2$ [$\sqrt{3}$ = 17.32]

$= 15.588 \, cm^2$.

EXAMPLE3:

Diagram I Shows a parallelogram *ABCD*. AX = 9 cm, *BC* = 14 cm and AY = 12.6 cm.

(a) Find the perimeter of the parallelogram

Diagram I

Diagram II

Diagram II shows a trapezium *PQRS*. *PS* = 12 cm and *QR* = 13 cm. The area of the trapezium is twice the area of the parallelogram.

(b) Calculate the height *PZ* of the trapezium

SOLUTION:

(a) Area of parallelogram $ABCD$

$= \text{Base} \times \text{Height}$

$= BC \times AX$

$= 9 \times 14$

$= 126 \text{ cm}^2$

Area of parallelogram $= 126 \text{ cm}^2$

$CD \times AY = 126$

$CD \times 12.6 = 126$

$\text{CD} = \frac{126}{12.6} = 10\ cm$

$= 10 \text{ cm}$

Perimeter of parallelogram

$= 2(BC + CD)$

$= 2(14 + 10)$

$= 48 \text{ cm}$

(b) Area of trapezium $= 2 \times 126 \text{ cm}^2$

$\frac{1}{2} \times \text{Sum of parallel sides} \times \text{Height} = 252$

$\frac{1}{2} \times (12 + 13) \times \text{PZ} = 252$

$\frac{1}{2} \times 25 \times \text{PZ} = 252$

$25\text{PZ} = 252 \times 2$

$\text{PZ} = \frac{504}{25}$

$= 20.16 \text{ cm}$

EXAMPLE 4:

The diagram shows three circles, each of radius 4 cm touching each other. A, B and C are the centres of the circles and D is the midpoint of AC. Taking π to be 3.142, calculate

(a) The perimeter of the figure,

(b) The area of the shaded region.

SOLUTION:

(a) Since ΔABC is an equilateral Δ

$\angle ABC = \angle BCA = \angle BAC = 60^0$

Reflex $\angle DAE = 360^0 - 60^0$ ($\angle$s at a point)

$= 300^0$

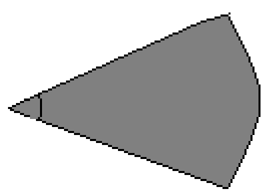
Perimeter of figure

$= 3 \times$ Length of major arc ADE

$= 3 \times \frac{300^0}{360^0} \times 2 \times 3.142 \times 4$ ⟵ Length of arc AB

$= 62.84$ cm

$= \frac{\theta}{360°} \times 2\pi r$



$= \frac{\theta}{360°} \times 2\pi r$

(b) Area of shaded region

$=$ Area of $\Delta ABC + 3 \times$ Area of major sector ADE

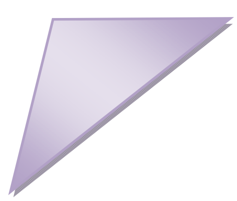
$= \frac{1}{2} \times 12 \times 12 \times \sin 60^0 + 3\ \frac{300^0}{360^0} \times 3.142 \times 6^2$

$\approx 345\ cm^2$ (correct to 3 sig. fig.)

TIPS FOR STUDENTS:

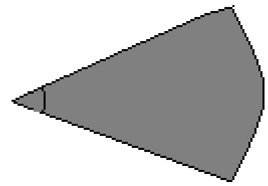
Area of triangle ABC

$= \frac{1}{2} \times bc \sin A$

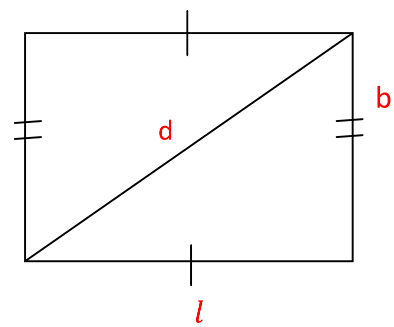


Ara of sector OAB

$= \frac{\theta}{360^0} \times \pi r^2$



1.3 PEREMETER AND AREA OF RECTANGLES:



1. Perimeter = Length of boundary

   = $2\,l + 2b = 2\,(\,l + b\,)$

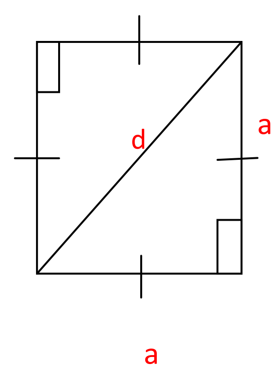
2. Area = Length X Breadth

   = $l^2 + b^2$

∴ Since, $d^2 = l^2 + b^2$ [Applying Pythagoras Theorem]

∴ Diagonal (d) = $\sqrt{l^2 + b^2}$

1.4 PEREMETER AND AREA OF SQUARES:



1. Perimeter = 4 a = 4 X side
2. Area = a X a = a^2 = $(\text{ sides })^2$
3. Diagonal (d) = $\sqrt{a^2 + a^2}$ = $\sqrt{2a^2}$ = $a\sqrt{2}$ = side $\sqrt{2}$

All Sides are equal in Square whereas Opposite Sides are equal in Rectangle.

EXAMPLE5:

The perimeter of a rectangle is 28 cm and its length is 8 cm. Find its :

a) Breadth b) Area c) Diagonal

SOLUTION:

a) Since, Perimeter = $2 (l + b)$

$\Rightarrow$ $28 = 2 (8 + b)$ $\Rightarrow$ b = 6cm

b) Area = $l \times b$ = 6 cm^2 = 48 cm^2

c) Diagonal (d) = $\sqrt{l^2 + b^2}$ $\sqrt{8^2 + 6^2}$ = 10 cm

EXAMPLE6:

The diagonal of square is 20 m . Find its :

a) Area b) Length of one side c) Perimeter

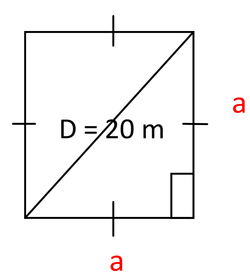
SOLUTION:

a) If each side of the square is a m ;

Then, $d^2 = a^2 + a^2$ [Pythagoras Theorem]

$\Rightarrow$ $(20)^2 = 2a^2$

$\Rightarrow$ $a^2 = \frac{400}{2} = 200$



∴ a) Area = a^2 = 200 m2

b) Since, a^2 = 200 $\Rightarrow$ a = $\sqrt{200}$ = 14.1 m

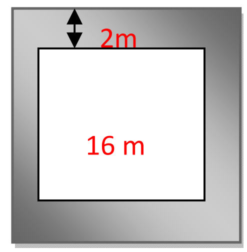
c) Perimeter = 4a = 4 X 14.1 m = 56.4 m

EXAMPLE6:

A path of uniform width 2m runs around the inside of a square field of side 20m. Find the area of the path.

SOLUTION:

According to the given information, the figure is as shown along side :



Clearly :

Area of the field including path = 20 m X 20 m = 400 m^2

Each side of the square field excluding path = (20 – 2 X 2) m = 16 m

$\therefore$ Area of the field excluding path = 16 m X 10 m = 256 m^2

$\therefore$ Area of the path = 400 m^2 – 256 m^2 = 144 m^2

EXAMPLE7:

A rectangular hall is 5.25 m long and 3.78 m wide. Its floor id to be covered with square tiles, each of side 21 cm, Find the cost of tiles required at the rate of Rs 5 per tile.

SOLUTION:

Since, Floor area of hall = 5.25 m X 3.78 m

= 525 X 378 cm^2

And, Area of each tile = 21 X 21 cm^2

Number of tiles required = $\frac{Floor\ area\ of\ hall}{Area\ of\ each\ tile}$

= $\frac{525\ X\ 378}{21\ X\ 21}$ = 450

Since, cost of each tile = Rs 5

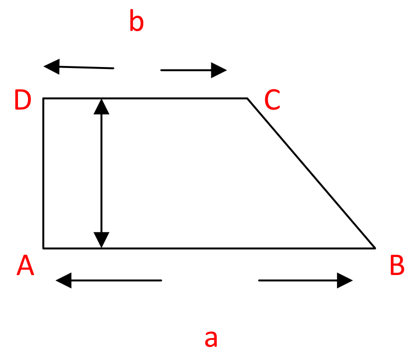
∴ The cost of all the tiles required = 450 X Rs 5 = Rs 2,250

1.5 TRAPEZIUM:

Area of a trapezium

= $\frac{1}{2}$ X (sum of parallel sides) X height

= $\frac{1}{2}$ X (a + b) X h



- Here, a and b are the parallel sides of the trapezium and h is the height .
- Height of the trapezium means the distance between its parallel sides .

EXAMPLE 8:

The length of parallel sides of a trapezium are in the ratio 3 : 5 and the distance between them is 10 cm . If the area of the trapezium is 120 cm^2, find the length of its parallel sides.

SOLUTION:

Let the length of parallel sides be 3x cm and 5x cm

Since, Area = $\frac{1}{2}$ (the sum of parallel sides) X height

$\Rightarrow$ 120 = $\frac{1}{2}$ (3x + 5x) X 10

$\Rightarrow$ 120 = 40 x ∴ x = $\frac{120}{40}$ = 3

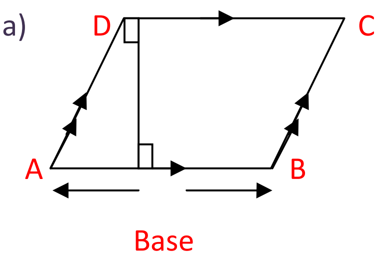
∴ Length of parallel sides = 3x cm and 5x cm

= 3 X 3 cm and 5 X 3 cm = 9 cm and 15 cm

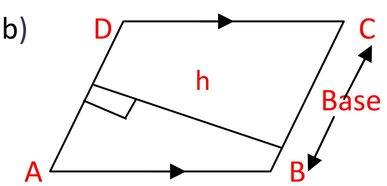
1.6 PARALLELOGRAM:

Area = Base X Corresponding height

e.g.- a)



e.g.- b)



For parallelogram ABCD, if side AB is taken as base, the corresponding height, is the distance between parallel sides AB and DC .

∴ Area = AB X *h*

And, if the side BC is taken as base, the corresponding height is the distance between parallel sides BC and AD .

∴ Area = BC X *h*

EXAMPLE9 :

A parallelogram has sides of 12 cm and 8 cm. If the distance between the 12 cm sides is 5 cm; find the distance between 8 cm sides .

SOLUTION:

According to the question ; if base = 12 cm; height = 5 cm

∴ Area = base X height = 12 X 5 cm^2 = 60 cm^2

Now, If base = 8cm; then to find height ?

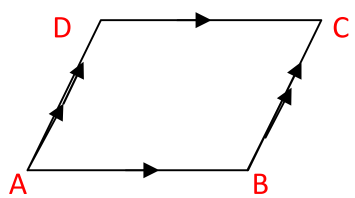
Area = base X height

⇒ 60 = 8 x h ⇒ $h = \frac{60}{8}$ = 7.5 cm

Each diagonal bisects a parallelogram ;

i.e. Δ ABC = Δ ADC = $\frac{1}{2}$ (//) gm ABCD)

Similarly, if diagonal BD is drawn :
Δ ABC = Δ BCD = $\frac{1}{2}$ (//) gm ABCD)



EXAMPLE10:

In a Parallelogram ABCD; AB = 16 cm, BC = 12 cm and diagonal AC = 20 cm. Find the area of the parallelogram.

SOLUTION:

For triangle ABC,

AB = 16 cm, AC = 20 cm and BC = 12 cm

∴ $s = \frac{a+b+c}{2}$ = $s = \frac{16+20+12}{2}$ = 24 cm

$\therefore$ Area of Δ ABC $= \sqrt{s\,(s-a)(s-b)(s-c)}$

$= \sqrt{24\,(24-16)(24-20)(124-12)}$

$= \sqrt{24 \times 8 \times 4 \times 12}$

$= \sqrt{9216}$

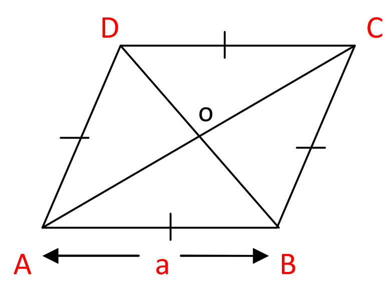
$= 96\ cm^2$

$\therefore$ Area Parallelogram ABCD = 2 X 96 cm^2 = 192 cm^2

1.7 RHOMBUS:

A Rhombus is a parallelogram whose all the sides are equal.

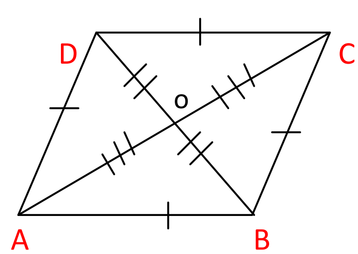
The adjoining figure shows a rhombus ABCD in which AB = BC = CD = DA = a



1. Perimeter of a rhombus = 4 X side = 4a

2. Area of a rhombus = $\frac{1}{2}$ X Product of its diagonals

$= \frac{1}{2}$ X AC X BD



- Diagonals of a rhombus bisect each other at right angle .
  $\therefore$ OA = OC = $\frac{1}{2}$ BD and $\angle$AOB = 90^0
- Δ AOB = Δ BOC = Δ COD = Δ DOA = $\frac{1}{4}$ X rhombus ABCD
- Since, a rhombus is a parallelogram :
  a) Each diagonal bisects it
  i.e. Δ ABC = Δ ADC = $\frac{1}{4}$ X rhombus ABCD

And, Δ ABD = Δ BCD = $\frac{1}{4}$ X rhombus ABCD

b) Area of rhombus = Base X Height

EXAMPLE11:

The diagonals if a rhombus are 16 cm 12 cm; find:

a) Its Area b) Length of its side c) Its Perimeter.

SOLUTION:

a) Area of rhombus = $\frac{1}{2}$ X Product of its diagonals

$$= \frac{1}{2} \times 16 \times 12\ cm^2 = 96\ cm^2$$

b) Given diagonal AC = 16 cm; then OA = OC = $\frac{16}{2}$ cm = 8 cm

Given diagonal BD = 12 cm; then OB = OD = $\frac{12}{2}$ cm = 6 cm

Since, the diagonals of a rhombus bisect at 90^0 ,

$\therefore$ Applying Pythagoras Theorem in Δ AOB ; we get :

$$(AB^2) = OA^2 + OB^2$$

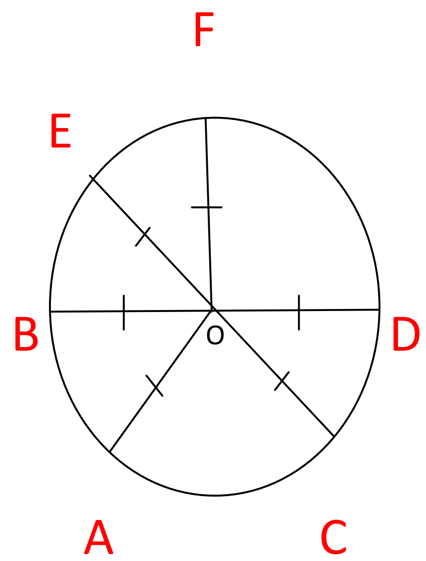
$$= 8^2 + 6^2 = 100$$

$$\therefore \quad AB = \sqrt{100} = 10\ cm$$

$\therefore$ Length of its side = 10 cm

c) Perimeter of rhombus = 4 X side = 4 X 10 = 4 0 cm

1.8 CIRCLE:



A CIRCLE is the surface enclosed by a close curve, obtained on joining all points that are at the same distance from a fixed point and lie in the same plane.

The adjoining figure shows a fixed point O and points A,B,D, ……, etc., that are at the same distance from fixed point O and lie in the same plane. The surface obtained on drawing a close curve through points A,B,C,D, …., etc., is the Circle shown by shaded portion .

The fixed point is called the centre and the constant distance is called the radius of the circle.Thus, point O is the centre and radius = OA = OB = OC = OD and so on .

The radius of a circle is in general represented by r.

1. DIAMETER:

A straight line, joining any two points on the circumference of the circle and passing through the centre, is called DIAMETER of the circle. Thus, BOD is a diameter.

Diameter = 2 x radius i.e. d = 2r.

2. CIRCUMFERENCE:

The *perimeter* of the circle is called its *Circumference*.

Tips for Students:

The Greek letter π (Pronounced as pie) represents the ratio between the circumference and diameter of a circle .

i.e. $\frac{Circumference\ of\ the\ circle}{Its\ diameter\ (d)} = \pi$

$\Rightarrow$ Circumference of the circle = πd

= $2\pi r$

3. AREA OF CIRCLE = πr^2 ; where π = 3.14 = $\frac{22}{7}$ and r = radius of the circle.

EXAMPLE12:

Find radius and area of a circle whose circumference is 132 cm .

SOLUTION:

Given Circumference = 132 cm

$\Rightarrow$ $2\pi r = 132$

$\Rightarrow$ $2 \times \frac{22}{7} \times r = 132$ $\Rightarrow$ $r = \frac{132 \times 7}{2 \times 22} = 21$ cm

$\therefore$ Area = $\pi r^2 = \frac{22}{7} \times (21)^2 = 1386\ cm^2$

EXAMPLE13:

Find circumference of the circle, whose area is 24.64 m^2

SOLUTION:

Given: πr^2 = 24 .64 m^2

$\Rightarrow \frac{22}{7} r^2 = 24.64$

$\Rightarrow r^2 = 24.64 \times \frac{7}{22} = 7.84 \Rightarrow r = 2.8$ m

$\therefore$ Circumference = $2\pi r = 2 \text{ X } \frac{22}{7} \text{ X } 2.8$ m = 17.6 m

EXAMPLE14:

The perimeter of a square, whose each side is 22 cm, is the same as circumference of a circle . Find area of the circle.

SOLUTION:

The perimeter of the square = 4 X its side

= 4 X 22 cm = 88 cm

$\Rightarrow$ Circumference of the circle = 88 cm

i.e., $2\pi r = \frac{22}{7} \text{ X } r$ = 88 cm

$\Rightarrow$ $2 = \frac{22}{7} \text{ X } r$ = 88 cm

And, $r = 88 \text{ X } \frac{7}{2 \text{ X } 22}$ cm = 14 cm

$\therefore$ Area of circle $= 2\pi r^2$

$= \frac{22}{7} \text{ X }$ 14 cm X 14 cm = 616 cm^2

EXAMPLE15:

The shaded portion in the adjoining figure shows a circular path enclosed by two concentric circle . If the inner circumference of the path is 176 m and the uniform width of the circular path is 3.5 m ; find the area of the path .

SOLUTION

Let the radius of the inner circle be r m

$\therefore \quad 2\pi r = 176 \Rightarrow 2 \times \frac{22}{7} \times r = 176$ m

$\Rightarrow \quad r = 176 \times \frac{7}{2 \times 22}$ m = 28 m

Since, the width of the path = 3.5 m

The radius of the outer circle (R) = 28 m + 3.5m = 31.5 m

The area of the circular path = Area of the outer circle – Area of the inner circle

$= \pi R^2 = \pi r^2$

$= (\frac{22}{7} \times 31.5 \times 31.5 - \times \frac{22}{7} \times 28 \times 28)\ m^2$

$= (3118.5 - 2464)\ m^2 = 654.5\ m^2$

> Whenever the value of π is not given, take : $\pi = \frac{22}{7}$

2.VOLUME AND SURFACE AREA OF SOLIDS:

2.1 BASIC CONCEPTS:

VOLUM SURFACE AREA	The space occupied by a body (solid) is called its volume. The sum of areas of all the faces of a body is called its surface area.	
UNIT OF LENGTH	UNIT OF VOLUME	UNIT OF SURFACE AREA
m (meter) cm mm	m^3 (cubic metre) cm^3 mm^3	m^2 (square metre) cm^2 mm^2

Also,

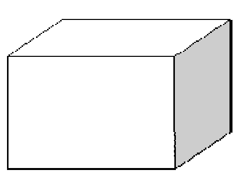
$1m^3 = 100 \text{ X } 100 \text{ X } 100\ cm^3 = 1000000\ cm^3$ and $1\ cm^3 = \frac{1}{100\ X\ 100\ x\ 100}\ m^3$

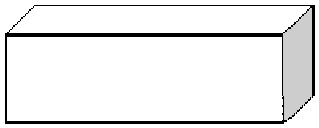
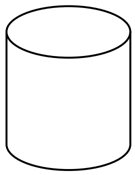
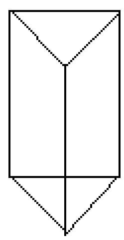
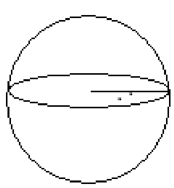
$1cm^3 = 10 \text{ X } 10 \text{ X } 10\ mm^3 = 1000\ mm^3$ and $1\ mm^3 = \frac{1}{10\times10\times10}\ cm^3$

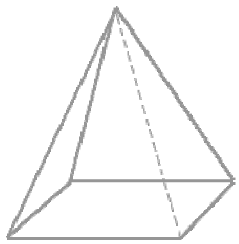
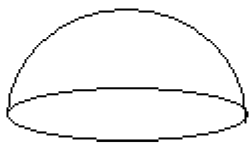
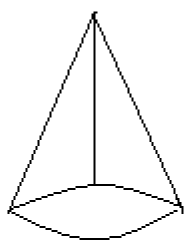
In general, the volume of a liquid or a gas is measured in litres, such that

$1\ m^3 = 1000$ liter and 1 liter = $1000\ cm^3$ (c.c. or millilitre).

2.2 The table below gives the formula to find the volume / surface area of some solids.

Solid	Formula
Cube 	Volume = s^3 Total surface area = $6s^2$ S = length of a side

Cuboid 	Volume = lbh Total surface area = $2(lb + lh + bh)$ l = length; b = breadth; h = height
Cylinder 	Volume = Base area × Height $= \pi r^2 h$ Total surface area = $2\pi r^2 + 2\pi rh$ r = base radius; h = height
Prism  Area of cross-section	Volume = Area of cross-section × Height Total surface area $= \begin{pmatrix}\text{Perimeter of} \\ \text{cross} - \text{section}\end{pmatrix} \times \text{Height} + 2 \times \begin{pmatrix}\text{Area of} \\ \text{cross} - \text{section}\end{pmatrix}$ or Volume = Area of cross-section × Length Total surface area $= \begin{pmatrix}\text{Perimeter of} \\ \text{cross} - \text{section}\end{pmatrix} \times \text{Length} + 2 \times \begin{pmatrix}\text{Area of} \\ \text{cross} - \text{section}\end{pmatrix}$
Sphere 	Volume = $\frac{4}{3}\pi r^3$ Surface area = $4\pi r^2$ r = radius
Pyramid	Volume = $\frac{1}{3}$ × Base area × Height Total surface area = Total area of all its faces = Base area + Area of all lateral faces

	
Hemisphere 	Volume = $\frac{2}{3}\pi r^3$ Curved surface area = $2\pi r^2$ Total surface area = $2\pi r^2 + \pi r^2$ $= 3\pi r^2$ r = radius
Cone 	Volume = $\frac{1}{3} \times$ Base area $\times$ Height $= \frac{1}{3}\pi r^2 h$ Curved surface area = $\pi r l$ Total surface area = $\pi r^2 + \pi r l$ r = base radius; h = height; l = slant height

EXAMPLE16:

The diagram shows a metal solid which is made up by joining a solid cone to a solid hemisphere. The diameter of the cone is 14 cm and the height of the solid is 31 cm.

(a) Find

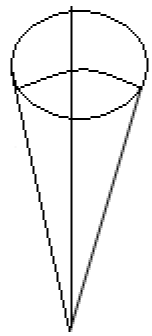
(i) The mass of the solid, in kilograms, if the density of the metal used is 6 g/cm^3,

(ii) The total surface area of the solid.

(b) The solid is melted down and recast into a square pyramid. If the height of the pyramid is 19 cm, find the length of a side of the square base.

[Take $\pi = \frac{22}{7}$]

SOLUTION:



(i) Radius of cone = 14 ÷ 2 = 7 cm

Radius of hemisphere = 7 cm

Height of cone = 31 – 7 = 24 cm

Volume of solid

= Volume of hemisphere + Volume of cone

$= \frac{2}{3} \times \frac{22}{7} \times 7^3 + \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 24$

$= 1950\frac{2}{3}\ \text{cm}^3$

Mass of solid

= Density × Volume

$= 6\ \text{g/cm}^3 \times \ = 1950\frac{2}{3}\ \text{cm}^3$

= 11 704 g

= 11.704 kg $\leftarrow 1\ kg = 1000g$

(ii) Let the slant height of the cone be *l* cm.

Using Pythagoras' Theorem,

$l^2 = 24^2 + 7^2$

= 625

$l = \sqrt{625}$

= 25 cm

Total surface area of solid

$= \left(\begin{matrix}\text{Curved surface} \\ \text{area of hemisphere}\end{matrix}\right) + \left(\begin{matrix}\text{Curved surface} \\ \text{area of cone}\end{matrix}\right)$

$= 2 \times \frac{22}{7} \times 7^2 + \frac{22}{7} \times 7 \times 25$

$= 858 \text{ cm}^2$

(c) Let the length of a side of the square base of the pyramid be *x* cm.

Volume of pyramid = Volume of solid

$\frac{1}{3} \times \text{Base area} \times \text{Height} = 1950\frac{2}{3} \text{ cm}^3$

$\frac{1}{3} \times x^2 \times 19 = 1950\frac{2}{3}$

$x^2 = \frac{1950\frac{2}{3} \times 3}{19}$

$= 308$

$x = \sqrt{308}$

$\approx$ 17.5 cm (correct to 3 sig. fig.)

∴ The length of a side of the square base is 17.5 cm.

EXAMPLE17:

Diagram I shows a tank which is completely filled with liquid. ABCD is the uniform cross-section of the tank which is trapezium. ADHE and BCGF are horizontal rectangles. AB = 4m, BC = 6m, AD = 10m, CG = 5m and $< ABC = 90^0$.

(a) Calculate the volume of liquid in the tank.

All the liquid in the tank is then poured into the container shown in diagram II. The container is completely filled with the liquid. The container is made by joining a cylinder of base radius 3m and height 4m to a cone of base radius 3m.

(b) Find the height of the cone.

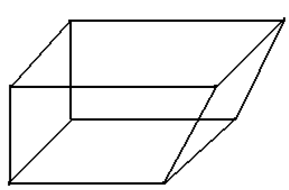


Diagram I

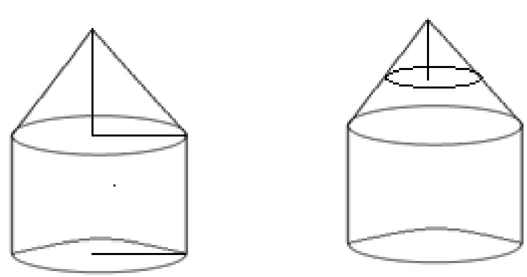


Diagram II

After some liquid is drained from the container, the height of the surface of the liquid below the tip of the container is 3.5m as shown in Diagram III.

(c) Calculate the volume of liquid left in the container.

[Take $\pi = 3.142$]

SOLUTION:

(a) Area of trapezium ABCD

$= \frac{1}{2} \times \text{Sum of parallel sides} \times \text{Height}$

$= \frac{1}{2} \times (6 + 10) \times 4$

$= 32\text{m}^2$

Volume of water in tank

$= \begin{pmatrix} \text{Area of trapezium} \\ ABCD \end{pmatrix} \times CG$

$= 32 \times 5$

$= 160 \text{ m}^3$

(b) Let the height of the cone be h m.

Volume of water in container = Volume of water in tank

$\begin{pmatrix} \text{Volume of} \\ \text{cylinder} \end{pmatrix} + \begin{pmatrix} \text{Volume of} \\ \text{cone} \end{pmatrix} = 160\text{m}^3$

$$3.142(3)^2(4) + \frac{1}{3}(3.142)(3)^2(h) = 160$$

$$113.112 + 9.426h = 160$$

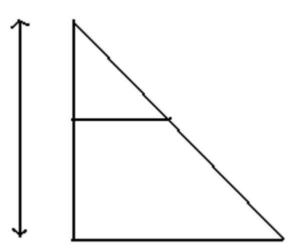
$$9.426h = 46.888$$

$$h = 4.974$$

$$\approx 4.97\text{m (correct to 3 sig. fig.)}$$

$\therefore$The height of the cone is 4.97 m

(c)



Using similar triangles,

$\frac{r}{3} = \frac{3.5}{4.974}$

$r = \frac{3.5}{4.974} \times 3$

$= 2.11m$

Volume of cone not occupied by liquid

$= \frac{1}{3} \times 3.142 \times 2.111^2 \times 3.5$

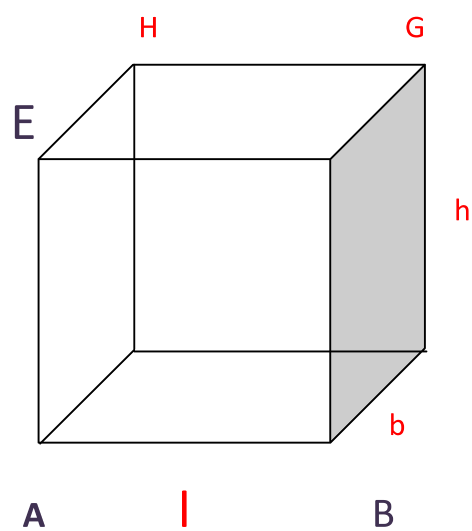
$= 16.335m^3$

Volume of liquid left in the container

$= 160 - 16.335$

$\approx 144\ m^3$ (correct to 3 sig. fig.)

2.3 CUBOID:(A RECTANGULAR SOLID)



A Cuboid is a solid bounded by six rectangular faces.

1. **VOLUME OF A CUBIOD**

= its length X breath X height

= *l* X *b* X *h*

2. **Total surface area of a Cuboid = Area of six rectangular faces.**

Since, Area of ABCD + Area of EFGH = 2 (*l* X *b*) [Opposite faces are equal]

Area of BCGF + Area of ADHE = 2 (*b* X *h*)[Opposite faces are equal]

And Area of ABFE + Area of DCGH = 2 (*h* X *l*) [Opposite faces are equal]

∴ Total surface area of Cuboid = 2 (*l* X *b* + *b* X *h* + *h* X *l*)

2.4 CUBE:

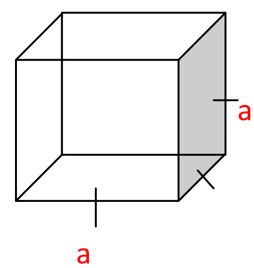
A cube is a rectangular solid whose each face is a square.

In other words , A Cube is a Cuboid whose length = breath = height = a

1. Since Volume of a cuboid = *l* X *b* X *h*

∴ Volume of a cube = a X a X a

= a^3 = $(\text{edge})^3$



EXAMPLE18:

The length, breath and height of a cuboid are in the ratio 6 : 5 : 4 . If its volume is 15,000 cm^2;

Find : a) Its dimensions b) Its surface area .

SOLUTION:

Dimension means: Its length, breadth and height.

a) Given : Length : Breadth : Height = 6 : 5 : 4

⇒ If length = 6x cm, breath = 5x cm and height = 4x cm

∴ Length : Breadth : Height = Volume

⇒ 6x X 5x X 4x = 15,000

$\Rightarrow$ $x^3 = \frac{15,000}{6 X 5 X 4} = 125 = 5 X 5 X 5 = 5^3$

$\therefore$ $x = 5$

i.e. Length = 6x cm = 6 X 5 cm = 30 cm

Breadth = 5x cm = 6 X 5 cm = 25 cm

And, Height = 4x cm = 4 X 5 cm = 20 cm

b) Surface area of the cuboid = 2 (*l* X *b* + *b* X *h* + *h* X *l*)

= 2 (30 X 25 + 25 X 20 + 20 X 30) cm^2

= 2 (30 X 25 + 25 X 20 + 20 X 30) cm^2

= 2 (750 + 500 + 600) cm^2

= 3700 cm^2

EXAMPLE19:

The total surface area of a cube is 294 cm^2 . find its volume .

SOLUTION:

Since Total surface area of cube = 6 X (side $)^2$

$\Rightarrow$ 6 x (side $)^2$ = 94

$\Rightarrow$ Side = 7 cm

$\therefore$ Volume = (side $)^3$ = (7 cm $)^3$ = 343 cm^3

EXAMPLE20:

A rectangular solid of metal has dimensions 50 cm. 64 cm and 72 cm. It is method and recast into identical cubes each with edge 4 cm, find the number of cube formed.

SOLUTION:

$\therefore$ Volume of rectangular solid melted = Its Length X Breath X height

= 50 X 64 X 72 cm^3

And, Volume of each cube formed = $(\text{ its edge })^3$

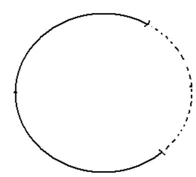
= $(4)^3\ cm^3 = 4 \times 4 \times 4 cm^3$

$\therefore$ Number of cubes formed = $\frac{Volume\ of\ solid\ melted}{Volume\ of\ each\ cube}$

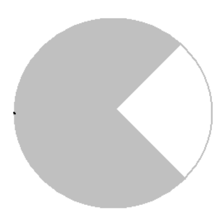
= $\frac{50 X 64 X 72}{4 X 4 X 4}$ = 3600 .

2.5 Arc length, Sector Area, Area of a Segment and Radian Measure:

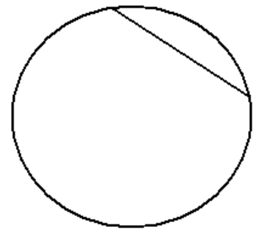
1. Arcs are part of the circumference of a circle. APB is the minor arc and AQB is the major arc.



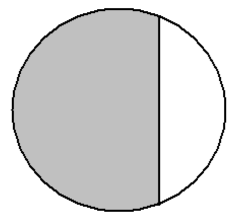
2.The part of a circle between two radii is called a sector. OAPB is the minor sector and OAQB is the major sector.



3. A chord is line segment that joins two points on a circle.



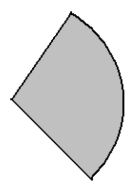
4. Segments are formed when chords divide the circle into different parts.



5. For a sector of a circle subtending an angle θ at the centre *O*, of radius r:

Length of arc AB

$$= \frac{\theta}{360^0} \times 2\pi r$$



Area of sector OAB

$$= \frac{\theta}{360^0} \times \pi r^2$$

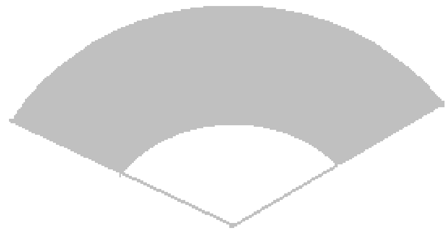
Where θ is in degrees.

EXAMPLE21:

The diagram shows the arcs PQ and RS of two circles with centre O. OR = 12cm, OP = PR and <ROS = 120^0.

(a) Calculate, in terms of π,
   (i) The perimeter of the shaded region,
   (ii) The area of the shaded region.

(b) Find the ratio of the area of the shaded region to the area of the sector ORS.

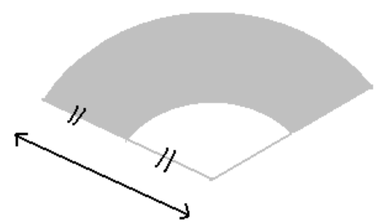


SOLUTION:

(a) (i)

OP = 12 ÷ 2 = 6cm

Perimeter of shaded region



= Length of arc PQ + Length of arc RS + PR + QS

$= \frac{120^0}{360^0} \times 2 \times \pi \times 6 + \frac{120^0}{360^0} \times 2 \times \pi \times 12 + 6 + 6$

$= 4\pi + 8\pi + 12$

$= 12\pi + 12$

$= 12(\pi + 1)$ cm

(iii) Area of shaded region

= Area of sector ORS – Area of sector OPQ

$= \frac{120^0}{360^0} \times \pi \times 12^2 - \frac{120^0}{360^0} \times \pi \times 6^2$

$= 48\pi - 12\pi$

$= 36\pi$ cm^2

(b) Required ratio

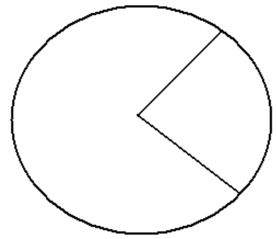
= Area of shaded region : Area of sector ORS

$= 36\pi : 48\pi$

= 3 : 4

Tips for students:

1. Two common units to measure angles are degrees and radians.
2. A radian is the measure of the angle subtended at the center of a cycle by an arc equal in length to the radius of the circle. ‘rad’ is the abbreviation for radian.



3. The formula to convert radians to degrees and vice-versa are given below.

π rad = 180^0
1 rad = $\frac{180}{\pi}$
$1^0 = \frac{\pi}{180} rad$

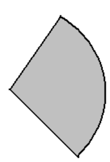
e.g.- $60^0 = 60 \times \frac{\pi}{180} = \frac{\pi}{3}\ rad \approx 1.05\ rad$ (correct to 3 sig. fig.)

$\frac{2\pi}{5} rad = \frac{2\pi}{5} \times \frac{180^0}{\pi} = 72^0$

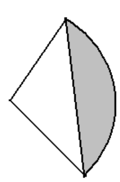
Arc Length, Area of a Sector and Area of a segment in Radian Measure:

For a sector of a circle, subtending an angle θ at the centre O, radius r:

Length of arc AB, s = rθ



Area of sector OAB, A = $\frac{1}{2}r^2\theta$



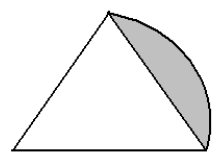
Area of segment APB

= Area of sector OAB – Area of ΔOAB

$= \frac{1}{2} r^2\theta - \frac{1}{2} r^2 \sin\theta$
Where θ is in radians.

EXAMPLE22:

In the diagram, OAB is a sector of a circle, centre *O* and radius 28 cm. Given that <AOB = 1.5 radians, calculate



(a) The length of the arc AB,
(b) The area of the sector OAB,
(c) The area of the shaded region.

SOLUTION:

(a) Length of arc AB = 28×1.5 $\leftarrow$ *take* $s = r\theta$
= 42 cm.

(b) Area of sector OAB = $\frac{1}{2} \times 28^2 \times 1.5$ $\quad \leftarrow$ *take* $A = \frac{1}{2}r^2\theta$
= 588 cm^2

(c) Area of shaded region
= Area of sector OAB – Area of ΔOAB
= $588 - \frac{1}{2} \times 28^2 \times \sin 1.5$

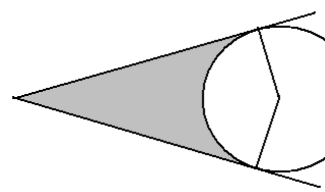
= 588 – 391.02
$\approx$ 197 cm^2

Tips for the students:

Do Switch your calculator to radian mode when using radian measure for trigonometric ratios.

EXAMPLE23:

In the diagram, O is the centre of the circle. TA and TB are tangents to the circle at A and B respectively. The radius of the circle is 12 cm and $<ATB = \frac{\pi}{5}$ radians.



Calculate:

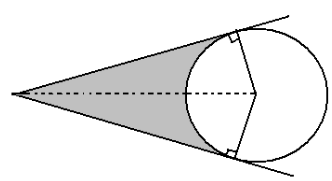
(a) The area of the shaded region,
(b) The perimeter of the shaded region.

SOLUTION:

(a) $< OAT = < OBT = \frac{\pi}{2}\, rad$ (tan ⊥ rad)

$< \text{ATO} = \frac{\pi}{5} \div 2 = \frac{\pi}{10}\ rad$

In ΔOAT,



$\tan\frac{\pi}{10} = \frac{12}{AT}$

$\text{AT} = \frac{12}{tan\frac{\pi}{10}}$

$= 36.93\text{cm}$

$<\text{AOB} = 2\pi - \frac{\pi}{5} - \frac{\pi}{2} - \frac{\pi}{2}$ $(\llcorner\, sum\ of\ quad.)$

$= \frac{4\pi}{5}\ rad$

Area of shaded region

$= 2 \times \begin{pmatrix} Area\ of \\ \Delta OAT \end{pmatrix} - \begin{pmatrix} Area\ of\ minor \\ sector\ OAB \end{pmatrix}$

$= 2 \times \left(\frac{1}{2} \times 36.93 \times 12\right) - \frac{1}{2} \times 12^2 \times \frac{4\pi}{5}$

$\approx 262\text{cm}^2$ (correct to 3 sig. fig.)

(b) Perimeter of shaded region

$= TA + TB + \text{Length of minor arc } AB$

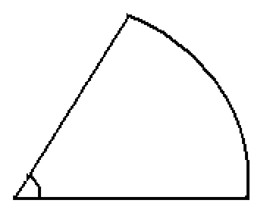
$= 2 \times 36.93 + 12 \times \frac{4\pi}{5}$

≈ 104cm (correct to 3 sig. fig.)

EXAMPLE24:

In the diagram, OAB is a sector of a circle with centre O. The length of the arc AB is $2\frac{1}{2}\pi$ cm

And the area of the sector OAB is $12\frac{1}{2}\pi\ cm^2$. Find the length of OA and $< AOB$ in degrees.



SOLUTION:

Let the radius of the sector be r cm and $< AOB$ be θ radians.

Length of arc AB = $2\frac{1}{2}\pi$cm (Given)

$$r\theta = 2\frac{1}{2}\pi \text{ ——— } (1)$$

Area of sector OAB $= 12\frac{1}{2}\pi cm^2$ (Given)

$$\frac{1}{2}r^2\theta = 12\frac{1}{2}\pi \text{ ——— } (2)$$

$$\frac{(2)}{(1)}: \quad \frac{\frac{1}{2}r^2\theta}{r\theta} = \frac{12\frac{1}{2}\pi}{2\frac{1}{2}\pi}$$

$$\frac{1}{2}r = 5$$

$$r = 10 \text{ cm}$$

2.6 APPLICATION:

1. FOR A ROOM:

Every room has four walls; two walls along its length and two walls along its width .

∴ a) Area of each wall along the Length = (l X h)

And, b) Area of each wall along the width = (b X h)

∴ Area of 4 walls of the room = (2 l X h + 2 b X h)

This Area Includes The Area Of Doors and Windows.

Also, c) Area of roof = Area of floor = l X b

EXAMPLE25 :

The internal length, breadth and height of a rectangular room are 6 m, 5.2 m and 4.5 m respectively. It has two doors each of 1.2 m by 2m and three windows each of 1 m by 80 cm.

Find the total internal area of the room to be white washed.

Also, Find the cost of whitewashing the room (excluding the doors and windows) at the rate of Rs 6 per m^2 .

SOLUTION:

For the room, its l = 6 m, b = 5.2 m and h = 4.5 m

∴ Area of its four walls = 2 (l + b) h

= 2 (6 + 5.2) X 4.5 m^2 = 100.8 m^2

Area of its roof = l X b = 6 X 5.2 m^2 = 31.2 m^2

Since, rea of one door = 1.2 X 2 m^2 = 2.4 m^2

∴ Area of two doors = 2 X 2.4 m^2 = 4.8 m^2

Also, Area of each window = 1 X 0.80 m2 = 0.80 m^2 [80 cm = 0.80 m]

∴ Area of each window = 3 X 0.80 m^2 – 2.40 m^2

∴ Total internal area of the room to be whitewashed

= (Area of four walls + Area of roof) – (Area of two doors + Area of three windows)

= (100.8 + 31.2) – (4.8 + 2.4) m^2

= 124.8 m^2

Cost of whitewashing = Rs 6 X 124.8 = Rs 748.80

2. FOR A BOX:

a) Space occupied by it = Its external volume

b) Its capacity = Its internal volume

c) Volume of material in it = Its external volume – Its internal volume

3. FOR A CLOSED BOX:

If its external Length, Breath and Height are l , b and h respectively, and its walls are x unit thick throughout, the :

a) Its Internal Length = External length – Twice the thickness of walls

= $l - 2x$

c) Its Internal Heigth = $h - 2x$

Conversely, if the internal dimensions of a box are l, b and h respectively and its sides are x unit thick everywhere, then its external dimensions are $l + 2x$, $b + 2x$ and $h + 2x$ respectively .

EXAMPLE27:

The external length, Breath and Height of a closed wooden box are 30 cm, 18 cm and 20 cm respectively . If the walls of the box are 1.5cm thick.

Find:

a) Capacity of the box

b) Volume of the wood used in making the box

And c) Weight of the box : If 1 cm^3 of the wood weighs 0.80 g.

SOLUTION:

Given, External length of the box = 30 cm

External breath of the box = 18 cm

And, External height of the box = 20 cm

∴ External volume of the box = 30 X 18 X 20 cm^3

= 10,800 cm^3

Since, the walls of the box are 1.5 cm thick throughout ;

∴ Internal length of the box = (30 – 2 X 1.5) cm = 27 cm

Internal breath of the box = (10 – 2 X 1.5) cm = 15 cm

And, Internal height of the box = (20 – 2 X 1.5) cm = 17 cm

∴ Internal volume of the box = 27 X 15 X 17 cm^3 ;

= 6,885 cm^3

a) Capacity of the box = Its internal volume

= 6,885 cm^3

b) Volume of the wood used = External volume - Internal volume

= 10,800 cm^3 – 6,885 cm^3

= 3,915 c m^3

c) Since , 1 cm^3 of wood weighs 0.80 g

$\therefore$ Weight of the box = 3,915 X 0.80g

= 3132 g = 3.132 kg

SUMMARY AND KEY POINTS

1. The perimeter of a plane figure is the total length of its boundary. The area of a plane figure is the amount of surface enclosed by its sides.

2. COMMON UNITS OF AREA ARE SQUARE METRE (Sq.m Or m^2), SQUARE CENTIMETRE (Sq.cm Or cm^2) , SQUARE MILLMETRE (Sq. mm Or mm^2), etc .

1 m^2 = 100 X 100 cm^2 = 10,000 cm^2 and 1 cm^2 = $\frac{1}{100 X 100}$ m^2 = 0.0001 m^2

1 dm^2 = 10 X 10 cm^2 = 100 cm^2 and 1 cm^2 = $\frac{1}{100}$ dm^2 = 0.01 dm^2

1 cm^2 = 10 X 10 mm^2 = 100 mm^2 and 1 mm^2 = $\frac{1}{100}$ cm^2 = 0.01 cm^2

3. COMMON UNITS OF PERIMETER ARE METRE (m),CENTIMETRE (cm),DECIMETRE (dm) etc .

- 1 m = 100 cm and 1 cm = $\frac{1}{100}$ m = 0.01 m
- 1 m = 10 dm and 1 dm = $\frac{1}{10}$ m = 0.1 m

1 m = 1000 cm and 1 mm = $\frac{1}{1000}$ m = 0.001 m and so on.

4. If a, b and c are the three sides of a triangle; then its

a) Perimeter = a + b + c

b) Area = $\sqrt{s\,(s-a)(s-b)(s-c)}$

where, s = semi – perimeter of the triangle = $\frac{a+b+c}{2}$

5. If one side (base) and the corresponding height (altitude)of the triangle are known, its

$$\text{Area} = \frac{1}{2}\ \text{base X height}$$

6. Also, Area = $\frac{1}{2}$ base X height $\Rightarrow$ a) Base = $\frac{2\ X\ Area}{Height}$

b) Height = $\frac{2\ X\ Area}{base}$

7. Perimeter of Rectangle = Length of boundary where l = length , b = breadth

= $2l + 2b = 2(l + b)$

Area = Length X Breadth

= $l^2 + b^2$

∴ Since, $d^2 = l^2 + b^2$

Diagonal (d) = $\sqrt{l^2 + b^2}$

8. Perimeter of a square = 4 a = 4 X side

Area = a X a = a^2 = $(\text{sides})^2$

Diagonal (d) = $\sqrt{a^2 + a^2}$ = $\sqrt{2a^2}$ = $a\sqrt{2}$ = side $\sqrt{2}$

9. Area of a trapezium

$= \frac{1}{2}$ X (sum of parallel sides) X height

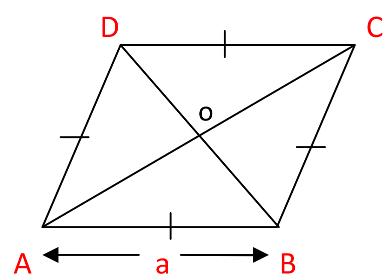
$= \frac{1}{2}$ X (a + b) X *h*

- Here, a and b are the parallel sides of the trapezium and h is the height .
- Height of the trapezium means the distance between its parallel sides .

10. Area of Parallelogram = Base X Corresponding height

11. A Rhombus is a parallelogram whose all the sides are equal.

The adjoining figure shows a rhombus ABCD in which AB = BC = CD = DA = a



Perimeter of a rhombus = 4 X side = 4a

Area of a rhombus = $\frac{1}{2}$ X Product of its diagonals

$= \frac{1}{2}$ X AC X BD

12. A CIRCLE is the surface enclosed by a close curve, obtained on joining all points that are at the same distance from a fixed point and lie in the same plane .

The fixed point is called the centre and the constant distance is called the radius of the circle.

a) DIAMETER: A straight line, joining any two points on the circumference of the circle and passing through the centre, is called DIAMETER of the circle.

Diameter = 2 x radius i.e. d = 2r.

b) CIRCUMFERENCE: The *perimeter* of the circle is called its *Circumference*.

Tips for students:

The Greek letter π (Pronounced as pie) represents the ratio between the circumference and diameter of a circle .

i.e. $\frac{Circumference\ of\ the\ circle}{Its\ diameter\ (d)} = \pi$

$\Rightarrow$ Circumference of the circle = πd

= 2 πr

c) AREA OF CIRCLE = πr^2 ; where π = 3.14 = $\frac{22}{7}$ and r = radius of the circle.

13. VOLUME AND SURFACE AREA (CUBOID AND CUBE):

The space occupied by a body (solid) is called its volume.

The sum of areas of all the faces of a body is called its surface area.

Tips for students:

In general, the volume of a liquid or a gas is measured in litres, such that

1 m^3 = 1000 liter and 1 liter = 1000 cm^3 (c.c.or millilitre)

14. A Cuboid is a solid bounded by six rectangular faces.

a) VOLUME OF A CUBIOD

= its length X breath X height

= *l* X *b* X *h*

b) Total surface area of a Cuboid = Area of six rectangular faces

Since, Area of ABCD + Area of EFGH = 2 (*l* X *b*) [Opposite faces are equal]

Area of BCGF + Area of ADHE = 2 (*b* X *h*) [Opposite faces are equal]

And Area of ABFE + Area of DCGH = 2 (*h* X *l*) [Opposite faces are equal]

$\therefore$ Total surface area of cuboid = 2 (*l* X *b* + *b* X *h* + *h* X *l*)

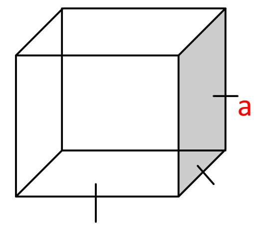
c) A Cube is a rectangular solid whose each face is a square.

In other words, a cube is a cuboid whose, length = breath = height = a

Since, Volume of a cuboid = *l* X *b* X *h*

∴ Volume of a cube = a X a X a

= a^3 = $(\text{edge})^3$



15. Dimension means:Its length, breadth and height.

a) For a Room:

Every room has four walls; two walls along its length and two walls along its width .

Area of 4 walls of the room = 2 (*l* X *h*) + 2 (*b* X *h*)

This Area Includes Area Of Doors and Windows.

Area of roof = Area of floor = $l \times b$

b) FOR a BOX:

Space occupied by it = Its external volume

Its capacity = Its internal volume

Volume of material in it = Its external volume – Its internal volume

c) FOR a CLOSED BOX:

If its external Length, Breath and Height are *l* , *b* and *h* respectively, and its walls are x unit thick throughout, then :

Its Internal Length = External length – Twice the thickness of walls

= *l* – 2x

Its Internal Heigth = *h* – 2x

If the internal dimensions of a box are *l*, *b* and *h* respectively and its sides are x unit thick everywhere, then its external dimensions are *l* + 2x, *b* + 2x and *h* + 2x respectively .